Deconfined Quantum Criticality and Conformal Phase Transition in Two-Dimensional Antiferromagnets

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Deconfined quantum criticality of two-dimensional $SU(2)$ quantum antiferromagnets featuring a transition from an antiferromagnetically ordered ground state to a so-called valence-bond solid state, is governed by a non-compact CP^1 model with a Maxwell term in 2+1 spacetime dimensions. We introduce a new perspective on deconfined quantum criticality within a field-theoretic framework based on an expansion in powers of $\epsilon = 4-d$ for fixed number N of complex matter fields. Namely, we show that in the allegedly weak first-order transition regime, a so-called conformal phase transition leads to a genuine deconfined quantum critical point. In a conformal phase transition, the gap vanishes when the critical point is approached from above and diverges when it is approached from below. We also find that the spin stiffness has a universal jump at the critical point.

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Many years have passed since a new paradigm for quantum phase transitions, the so-called deconfined quantum criticality (DQC) scenario, was introduced [1]. In this new paradigm, the effective quantum field theory does not contain any *elementary* fields representing the order parameters associated with the underlying competing orders. Rather, it is assumed that in certain quantum phase transitions these order parameters are not elementary, being themselves composed of more elementary fields in the same way that in elementary particle physics mesons are constituted by quarks. The precise context where this happens involves competing orders featuring broken *internal* and spacetime symmetries. This occurs, for example, in certain $SU(2)$ quantum antiferromagnets (AF) where $SU(2)$-invariant interactions compete. A paradigmatic example is the so-called $J-Q$ model [2],

$$H = J\sum_{\langle i,j\rangle}\mathbf{S}_i\cdot\mathbf{S}_j - Q\sum_{\langle ijkl\rangle}\left(\mathbf{S}_i\cdot\mathbf{S}_j-\frac{1}{4}\right)\left(\mathbf{S}_k\cdot\mathbf{S}_l-\frac{1}{4}\right), \tag{1}$$

where both J and Q are positive. Defining the dimensionaless coupling $g = Q/J$, we obtain the schematic phase diagram shown in Fig. 1. For $g \ll 1$ the first term in (1) dominates, favoring a Néel state. For $g \gg 1$ the plaquette term in Eq. (1) dominates, favoring a valence-bond solid (VBS) ordered state. The Néel state breaks an internal symmetry, namely $SU(2)$. The VBS state preserves the $SU(2)$ symmetry while breaking the symmetries of the square lattice. As one broken symmetry is internal [the $SU(2)$ one] and the other one is a spatial one, quantum mechanics forbids their coexistence, since the VBS state is a long-range entangled state while the Néel state is long-range ordered. Making the conventional assumption that both order parameters are the most elementary objects, an analysis using the Landau-Ginzburg-Wilson approach would in principle allow for the coexistence of the Néel and VBS states. Furthermore, such an analysis would invariably imply a first-order phase transition. The DQC scenario circumvents this, because the operators measuring both Néel and VBS order are comprised of more fundamental objects. These are the spinons, which are represented by an $SU(2)$ doublet of complex fields $\mathbf{z} = (z_1, z_2)$ satisfying the constraint $|z_1|^2 + |z_2|^2 = 1$ at each lattice point. Geometrically, this constraint represents the sphere S_3, which is the manifold associated with the $SU(2)$ Lie group. In terms of the spinon fields, the fields representing the Néel and VBS order parameters are $U(1)$ gauge-invariant objects. Thus, the effective spinon theory is naturally represented in terms of an Abelian gauge theory having a global $SU(2)$ symmetry [1]. The gauge field arising in such a theory is an emergent "photon". Moreover, the $U(1)$ gauge field is originally defined on the lattice, and hence it is necessarily compact. This leads to magnetic monopole excitations that gap the dual of the photon, defined as $B_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$ even in the phase where the expectation value of the Higgs field is zero, corresponding to the paramagnetic phase. This gap is the same as the mass of the magnetic monopoles [3]. The VBS phase is one where the spinons are confined (see the discussion in the caption of Fig. 1). Thus, the emergent photon is gapped in both the Néel and VBS phases, but in two different ways. One of the most fundamental predictions of the DQC scenario is that the mass of the dual photon continuously vanishes for g approaching a quantum critical point g_c from above, thus suppressing the magnetic monopoles at the quantum critical point [1]. The precise mechanism for this is destructive interference between the staggered Berry phase of the quantum AF and the magnetic monopoles. The Berry phase suppresses the monopoles as the phase transition to the Higgs (Néel) phase is approached. For a version of this theory with easy-plane anisotropy, the

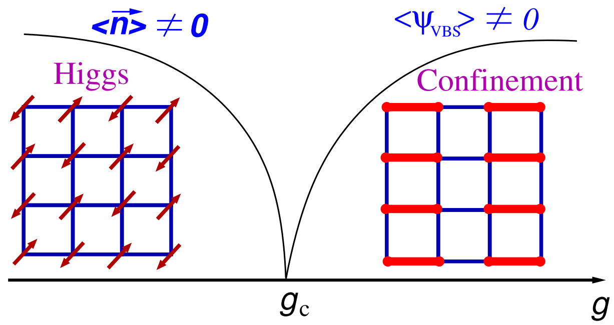

FIG. 1. Schematic phase diagram for the $J-Q$ model [Eq. (1)] showing a quantum phase transition between a Néel state and a VBS as a function of the dimensionless coupling $g = Q/J$. Both the Néel and the VBS order parameters are composed of spinon fields. On the lattice, they correspond to the composite fields $\mathbf{n}_i = (-1)^i z^*_{i\alpha}\boldsymbol{\sigma}_{\alpha\beta} z_i$ and to $\psi_{\mathrm{VBS},i} = (-1)^i L^{-1}\sum_j z^*_{i\alpha} z_{j\alpha} z^*_{j\beta} z_{i\beta}$, where L is the number of lattice sites. In terms of the spinons $z_{i\sigma}$ both fields represent $U(1)$ gauge-invariant operators. In the Higgs phase, the spinons condense due to a spontaneous $U(1)$ symmetry breaking, leading to a Néel state. In the confinement phase, all excitations are gapped and the spinons are confined, leading to a VBS state. In the Néel phase the emergent photon is gapped, while in the confined phase it is the dual of the emergent photon of the Higgs phase which is gapped.

suppression of magnetic monopoles has been confirmed by Monte Carlo (MC) simulations [4]. In the easy-plane case, the suppression occurs in a weak first-order phase transition, and no quantum criticality ensues [4, 5].

For the $SU(2)$ model, early MC results indicated a weak first-order phase transition [7]. Simulations performed on the $J-Q$ model have mostly yielded a second-order phase transition and strong signs of an emergent $U(1)$ symmetry [2, 8, 9], although a weak first-order phase transition has also been reported [10]. Since the $J-Q$ model is one of the emblematic lattice models for the DQC scenario, a recent MC study [11] made a comparative analysis of its phase diagram with the one obtained from the non-compact CP^1 model. While both models agree over a substantial portion of the phase diagram for moderate system sizes, they behave differently at larger system sizes [11]. Furthermore, there are indications that none of the models become critical, which would corroborate a weak first-order phase transition scenario. Recent large scale simulations [12] on the non-compact Abelian Higgs model with CP^1 constraint indicate that the existence of a tricritical point cannot be ruled out.

This brings us to the main topic of this paper, namely, a quantum field-theoretic analysis of the non-compact Abelian Higgs model with a global $SU(N)$ symmetry. In the present context, there are two relevant versions of this theory, a non-linear and a linear one. The non-linear theory corresponds to a CP^{N-1} model with a Maxwell term [13],

$$\mathcal{L}_{\mathrm{CP}^{N-1}} = \frac{\Lambda^{d-2}}{\hat{g}}\sum_{\alpha=1}^{N}|(\partial_\mu - iA_\mu)z_\alpha|^2 + \frac{1}{4e^2}F^2_{\mu\nu}, \quad (2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as usual and the constraint $\sum_{\alpha=1}^{N}|z_\alpha|^2 = 1$ holds. The linear version softens this constraint and has the more standard Higgs model form [1]

$$\mathcal{L}_{\mathrm{Higgs}} = \sum_{\alpha=1}^{N}\left[|(\partial_\mu - iA_\mu)z_\alpha|^2 + r|z_\alpha|^2\right] + \frac{u}{2}\left(\sum_{\alpha=1}^{N}|z_\alpha|^2\right)^2 + \frac{1}{4e^2}F^2_{\mu\nu}. \quad (3)$$

Both models have the same symmetries. In parameter regimes where a critical point exists, they should belong to the same universality class. This can be conveniently investigated by considering the large N limit of both models. Indeed, in the limit $e^2 \to \infty$, both $\mathcal{L}_{\mathrm{CP}^{N-1}}$ and $\mathcal{L}_{\mathrm{Higgs}}$ have exactly the same critical behavior for large N [14]. However, a recent calculation of the spin stiffness at large N and finite e^2 [15] showed that ρ_s exponentiates to a Josephson scaling form only when $e^2 \to 0$ or $e^2 \to \infty$. This corresponds to the $O(2N)$ or CP^{N-1} universality classes, respectively. For finite values of e^2, the spin stiffness exhibits logarithmic violations of scaling [15]. Interestingly, such logarithmic violations of scaling in the spin stiffness have been reported in recent MC simulations of the $J-Q$ model [9, 16].

We start by briefly revisiting the ϵ-expansion of the Higgs model (3) and introduce the renormalized dimensionless couplings $f = m^{-\epsilon}e_R^2$ and $g = m^{-\epsilon}u_R$, where m is the renormalized Higgs mass, which corresponds to the inverse correlation length. The one-loop RG β functions are well known [17],

$$\beta_f \equiv m\frac{df}{dm} = -\epsilon f + \frac{N}{3}f^2, \quad (4)$$

$$\beta_g \equiv m\frac{dg}{dm} = -\epsilon g - 6fg + (N+4)g^2 + 6f^2. \quad (5)$$

There are two relevant regimes where critical points arise, depending on the value of the gauge coupling fixed point. For $f = 0$ we have a nontrivial fixed point $g_* = \epsilon/(N+4)$ governing the critical behavior corresponding to the $O(2N)$ universality class, while the line $f = f_* = 3\epsilon/N$ contains a critical (g_+) and a tricritical (g_-) fixed point for $N > N_c = 6(15 + 4\sqrt{15})$, given by $g_\pm = \epsilon(18 + N \pm \sqrt{\Delta})/[2N(N+4)]$, where $\Delta = N^2 - 180N - 540$. It is easily seen why leading order $1/N$ calculations fail in producing a tricritical point. Namely, for large N we have $g_+ \approx \epsilon/N$ and $g_- \approx 0$, implying that the tricritical point is a higher order effect in $1/N$. We are interested

in analyzing the quantum critical behavior near the line $f = f_*$. This corresponds to a regime where the bare gauge coupling becomes essentially infinite. This can be easily seen by considering the solution to Eq. (4),

$$f(m) = \frac{f_\Lambda (m/\Lambda)^{-\epsilon}}{1 + \frac{N f_\Lambda}{3\epsilon}[(m/\Lambda)^{-\epsilon} - 1]}, \tag{6}$$

where Λ is an ultraviolet cutoff, and $f_\Lambda = f(\Lambda) = e^2 \Lambda^{-\epsilon}$ is a finite number for large Λ. We find $f \to f_*$ as $e^2 \to \infty$, which is the same as $m \to 0$. We therefore see that the existence of a critical point is equivalent to having a strongly coupled theory. Hence, the behavior near the line $f = f_*$ should correspond to a crossover to the critical behavior of the CP^{N-1} model (2). In order to understand this quantum critical behavior, we solve Eq. (5) along the line $f = f_*$ for $N > N_c$, obtaining

$$\frac{m^2}{\Lambda^2} = \left(\frac{g_+ - g}{g - g_-}\right)^{2/\omega}, \tag{7}$$

where $\omega = m\partial\beta_g(g_+, f_*)/\partial m = \epsilon\sqrt{\Delta}/N$ is the exponent governing corrections to scaling. Note that $g \to g_+$ for $m \to 0$, while $g \to g_-$ when $m^2 \gg \Lambda$.

Next, consider the case $N < N_c$. We are ultimately interested in the case $N = 2$. In this case, the fixed points $g_\pm$ are complex conjugate to each other, since $\Delta < 0$. In this case m does not have a power-law behavior. Rather, we obtain,

$$\frac{m^2}{\Lambda^2} = \exp\left\{-\frac{2N}{\epsilon\sqrt{|\Delta|}} \arctan\left[\frac{\sqrt{|\Delta|}\epsilon}{2N(N+4)(g - g_c)}\right]\right\}, \tag{8}$$

where $g_c = \mathrm{Re}\ (g_\pm)$. The limit $\epsilon \to 0$ features only a Gaussian fixed point. In this case, it is known that the phase transition is first-order [18]. In the limit $\epsilon \to 0$, both Eqs. (7) and (8) become $m^2/\Lambda^2 = \exp\{-2/[(N+4)g]\}$. The CP^{N-1} model has a similar behavior at its critical dimension, $d = 1+1$. We note that m^2/Λ^2 does not vanish at $g = g_c$. As $g \to g_c+$ it approaches its minimum value, $(m^2/\Lambda^2)_{\min}$, and jumps abruptly to its maximum value, $(m^2/\Lambda^2)_{\max}$, which is attained as $g \to g_c-$. The difference $(m^2/\Lambda^2)_{\max} - (m^2/\Lambda^2)_{\min}$ is much larger than $(m^2/\Lambda^2)_{\min}$, showing that the mass gap fails to vanish as g_c is approached from above. Adhering further to the logic of the ϵ-expansion, we can write approximately,

$$\frac{m^2}{\Lambda^2} \approx \exp\left[-\frac{1}{(N+4)(g - g_c)}\right], \tag{9}$$

which vanishes as $g \to g_c+$. On the other hand, approaching g_c from below causes m to grow to infinity. Theories with this type of behavior featuring an essential singularity, are said to undergo a conformal phase transition (CPT) [19]. A CPT corresponds to a higher dimensional analog of the Berezinsky-Kosterlitz-Thouless (BKT) transition [20], in which the inverse correlation length features an essential singularity at the critical point. CPTs frequently occur in gauge theories, and are associated with a breakdown of conformal symmetry at a quantum critical point. This aspect of gauge theories is related to the so called trace anomaly [21] of the stress tensor. In the present context, it can be shown that the trace θ of the stress tensor, does not vanish at the critical point. Rather, $\theta = (\eta_A/4)F_{\mu\nu}^2$, where $\eta_A = \epsilon$ is the anomalous dimension of the photon. When $\epsilon = 0$, the theory is conformally invariant at the fixed point, the first-order character of the phase transition not withstanding. This is because at the fixed point in $d = 3+1$, the theory corresponds to a free field theory.

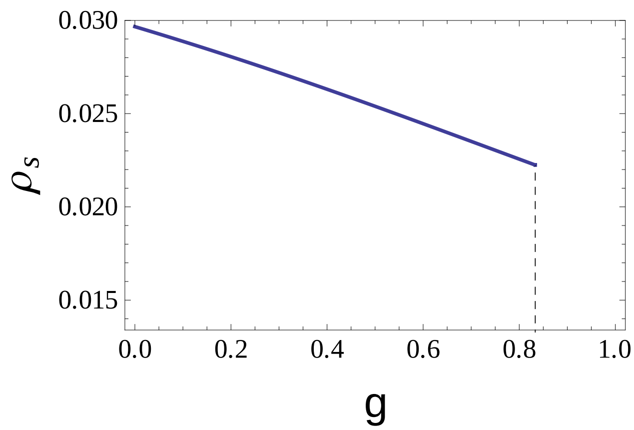

FIG. 2. Spin stiffness obtained by inserting Eq. (8) into Eq. (11) and setting $N = 2$ and $\epsilon = 1$. There is a universal jump at $g = g_c$.

It is also interesting to note a further parallell between the BKT transition and a CPT. In gauge theories, Elitzur's theorem [22] forbids the spontaneous breaking of a local gauge symmetry in any dimension, while in a BKT transition the Mermin-Wagner theorem [23] excludes spontaneous symmetry breaking of a global continuous symmetry in two dimensions. Recent lattice simulations [24] shows evidence for the existence of a CPT in $SU(N)$ gauge theories in $d = 3+1$. The result (9) indicates that the phase transition associated with DQC quite possibly may not be rubricated as a first- or second-order phase transition.

In order to find further signatures of a CPT, we search for universal behavior in physical quantities. The spin stiffness ρ_s is a crucial physical observable in DQC. In the case of a CPT, it must have a behavior similar to what is found in a BKT transition, where the superfluid stiffness exhibits a universal jump at the critical point [25].

To facilitate computing ρ_s within the present formalism, we observe that in the Higgs phase the renormalized photon mass is given by $m_A^2 = 2e_R^2\rho_s$ and use the fact that $m^2/m_A^2 = g/(2f)$ to derive an RG equation for ρ_s, $md\rho_s/dm = (2 - \epsilon - \beta_g/g)\rho_s$, and solve it

over the line $f = f_*$. The solutions have the scaling form, $\rho_s = m^{2-\epsilon}F(m/\Lambda)$. Consider first the case having $N > N_c$, where a second-order phase transition takes place. We obtain the typical Josephson scaling, including corrections to scaling behavior

$$\rho_s = \frac{m^{2-\epsilon}[1+(m/\Lambda)^\omega]}{N+18+\sqrt{\Delta}+(N+18-\sqrt{\Delta})(m/\Lambda)^\omega}. \quad (10)$$

When $N < N_c$, on the other hand, we have

$$\rho_s = \frac{m^{2-\epsilon}}{N+18+\sqrt{|\Delta|}\tan\left[\frac{\epsilon}{2N}\sqrt{|\Delta|}\ln\left(\frac{m}{\Lambda}\right)\right]}. \quad (11)$$

The factor ϵ in the argument of the tangent in Eq. (11) prevents the use of Eq. (9) in it, and Eq. (8) has to be used instead. This leads to a universal jump as $g \to g_c+$ given by

$$\frac{\rho_s^c}{\Lambda^{2-\epsilon}} = \frac{\exp\left[\frac{N\pi(2-\epsilon)}{2\epsilon\sqrt{|\Delta|}}\right]}{N+18+\sqrt{|\Delta|}}. \quad (12)$$

Thus, we have obtained another expected feature of a CPT reminiscent of the BKT behavior [25].

Note that when expressed in terms of m, Eq. (11) exhibits a logarithmic violation of scaling, a behavior observed in MC simulations of the $J-Q$ model [9, 16] and discussed recently in a large N context in Ref. [15]. In Fig. 2 we plot ρ_s for $N = 2$ and $\epsilon = 1$.

As a final calculation to support a CPT scenario in DQC, we consider the dynamics of magnetic monopoles inside the VBS. In order to analyze it, we need to perform the calculation at fixed dimensionality. A controlled way of doing this is by using the large N formalism in the CP^{N-1} model, Eq. (2). At leading order a standard calculation yields the mass gap, M, for $\hat{g} > \hat{g}_c$,

$$\frac{M^2}{\Lambda^2} = \left[\frac{2}{\pi}\left(1-\frac{\hat{g}_c}{\hat{g}}\right)\right]^2, \quad (13)$$

where $\hat{g}_c = 2\pi^2/N$. By computing the vacuum polarization, we obtain the correction to the Maxwell term responsible for the most important contribution to the monopole dynamics. By considering the strong-coupling regime $e^2 \to \infty$, we obtain

$$\mathcal{L}_{\rm Maxwell} \approx \frac{N}{48\pi M}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2. \quad (14)$$

A Maxwell Lagrangian in three spacetime dimensions supports magnetic monopoles, provided the $U(1)$ gauge group is compact. This amounts to considering compact electrodynamics [3], which is equivalent to a field theory for a Coulomb gas of monopoles. The monopole action is given by

$$S_{\rm inst} = \frac{N}{48M}\sum_{i\neq j}\frac{q_i q_j}{|x_i - x_j|} + \frac{N\Lambda}{24\pi M}\sum_i q_i^2 - 2N\sum_i \rho_{q_i}\ln\left(\frac{M}{\Lambda}\right), \quad (15)$$

where $q_i = \pm1, \pm2, \ldots$. The first two terms are the usual contributions originating with compact electrodynamics in $2+1$ dimensions [3]. The last term was computed in Ref. 26. We will consider only the contribution having instanton charges $q_i = \pm1$, which yields $\rho_1 \approx 0.06$ [26]. Therefore, the corresponding field theory for the monopoles is given by the following sine-Gordon Lagrangian

$$\mathcal{L}_{\rm SG} = \frac{1}{2}(\partial_\mu\varphi)^2 - z\cos(2\pi s\varphi), \quad (16)$$

where $s = \sqrt{N\hat{g}/[48\Lambda(\hat{g}-\hat{g}_c)]}$, and

$$z = \frac{\Lambda^3}{(\hat{g}-\hat{g}_c)^{2N\rho_1}}\exp\left[-\frac{N\hat{g}}{48(\hat{g}-\hat{g}_c)}\right], \quad (17)$$

is the fugacity of the Coulomb gas. Therefore, within a Debye-Hückel approximation, we find the screening mass gap of the monopole gas given by

$$M_{\rm DH}^2 = 4\pi^2 r^2 z = \frac{\Lambda^2\pi^2 N\hat{g}}{12(\hat{g}-\hat{g}_c)^{1+2N\rho_1}}\exp\left[-\frac{N\hat{g}}{48(\hat{g}-\hat{g}_c)}\right]. \quad (18)$$

Eq. (18) gives the mass gap of the confining dual photon in the VBS phase. Note that this is not a simple power law, providing further indications of a CPT.

Summarizing, to obtain improved understanding of deconfined quantum criticality, we have analyzed the ϵ-expansion of the Abelian Higgs model in the allegedly first-order phase transition regime along a line in the RG flow diagram determined by the gauge coupling fixed points defining the strong-coupling regime. We have argued that within the accuracy of the ϵ-expansion, a conformal phase transition associated with a deconfined quantum critical point occurs. We obtain a spinon mass gap featuring an essential singularity at the critical point. Similarly to the BKT transition in two dimensions, we find that the spin stiffness has a universal jump at the conformal phase transition critical point. We find further evidence for a conformal phase transition by analyzing the VBS phase at large N in the presence of magnetic monopoles, where the screening mass of the monopoles also exhibits an essential singularity at the critical point.

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[1] T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, and M. P. A. Fisher, Science, **303**, 1490 (2004); T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Phys. Rev. B **70**, 144407 (2004).
[2] A. W. Sandvik, Phys. Rev. Lett. **98**, 227202 (2007).
[3] A. M. Polyakov, Nucl. Phys. B **120**, 429 (1977).
[4] S. Kragset, E. Smørgrav, J. Hove, F. S. Nogueira, and A. Sudbø, Phys. Rev. Lett. **97**, 247201 (2006).
[5] A. B. Kuklov, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Ann. Phys. (N.Y.) **321**, 1602 (2006).
[6] O. I. Motrunich and A. Vishwanath, Phys. Rev. B **70**, 075104 (2004).
[7] A. B. Kuklov, M. Matsumoto, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, Phys. Rev. Lett. **101**, 050405 (2008).
[8] R. G. Melko and R. K. Kaul, Phys. Rev. Lett. **100**, 017203 (2008).
[9] A. W. Sandvik, Phys. Rev. Lett. **104**, 177201 (2010).
[10] F.-J. Jiang, M. Nyfeler, S. Chandrasekharan, and U.-J. Wiese, J. Stat. Mech. P02009 (2008).
[11] K. Chen, Y. Huang, Y. Deng, A. B. Kuklov, N. V. Prokof'ev, and B. V. Svistunov, arXiv:1301.3136.
[12] E. V. Herland, T. A. Bojesen, E. Babaev, and A. Sudbø, Phys. Rev. B **87**, 134503 (2013).
[13] I. D. Lawrie and C. Athorne, J. Phys. A: Math. Gen. **16**, L587 (1983); *Ibid.* **16**, 4428 (1983).
[14] S. Hikami, Prog. Theor. Phys. **62**, 226 (1979).
[15] F. S. Nogueira and A. Sudbø, Phys. Rev. B **86**, 045121 (2012).
[16] A. Banerjee, K. Damle, and F. Alet, Phys. Rev. B **82**, 155139 (2010).
[17] B. I. Halperin, T. C. Lubensky, and S.-K. Ma, Phys. Rev. Lett., **32**, 292 (1974).
[18] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[19] V. A. Miransky and K. Yamawaki, Phys. Rev. D **55**, 5051 (1997).
[20] V.L. Berezinsky, Sov. Phys. JETP **32**, 493 (1971) [Zh. Eksp. Teor. Fiz. **59**, 907 (1970)]; J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[21] J. C. Collins, A. Duncan, and S. D. Joglekar, Phys. Rev. D **16**, 438 (1977); S. D. Joglekar and A. Misra, Phys. Rev. D **38**, 2546 (1988); J. Polchinski, Nucl. Phys. B **303**, 226 (1988).
[22] S. Elitzur, Phys. Rev. D **12**, 3978 (1975).
[23] N. D. Mermin and H. Wagner, Phys. Rev. Lett., **22**, 1133 (1966).
[24] A. Deuzeman, M. P. Lombardo, and E. Pallante, Phys. Rev. D **82**, 074503 (2010).
[25] D. R. Nelson and J. M. Kosterlitz, Phys. Rev. Lett., **39**, 1201 (1977).
[26] G. Murthy and S. Sachdev, Nucl. Phys. B **344**, 557 (1990).